

Formation of Giant Planets by Concurrent Accretion of Solids and Gas inside an Anti-Cyclonic Vortex

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ABSTRACT

We study the formation of a giant gas planet by the core-accretion gas-capture process, with numerical simulations, under the assumption that the planetary core forms in the center of an anti-cyclonic vortex. The presence of the vortex concentrates particles of centimeter to meter size from the surrounding disk, and speeds up the core formation process. Assuming that a planet of Jupiter mass is forming at 5 AU from the star, the vortex enhancement results in considerably shorter formation times than are found in standard core-accretion gas-capture simulations. Also, formation of a gas giant is possible in a disk with mass comparable to that of the minimum mass solar nebula.

Subject headings: accretion, accretion disks — circumstellar matter — hydrodynamics — instabilities — turbulence — methods: numerical — solar system: formation — planetary systems

1. Introduction

Two different models for the formation of gas giant planets in a disk of gas and dust are under discussion. Both are subject to major difficulties. A gravitational instability in a

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disk (Boss 1997) would be a fast process but probably is not achievable, except in the outer regions of a disk (beyond ≈ 30 AU; see §2.1), because of the requirement that the cooling rate by radiation be comparable to or shorter than the orbital time scale (Gammie 2001; Rice et al. 2003). The core accretion model, on the other hand, explains a number of observed properties of the giant planets in the solar system, but for standard low-mass disk parameters it requires a time scale that is longer than the expected lifetime of the solar nebula. In order to explain the formation of Jupiter within 3 million years or less (Hubickyj, Bodenheimer, & Lissauer 2005), one has to postulate an enhancement in the surface density of solid particles in the disk well above (by a factor of ~ 3) that in the minimum mass solar nebula (MMSN; Weidenschilling 1977, Hayashi 1981). Other detailed studies of the accretion of the core of Jupiter (Weidenschilling 1998; Inaba, Wetherill, & Ikoma 2003; Thommes, Duncan, & Levison 2003) suggest that the disk has to have up to 10 times the mass of the MMSN. Formation of the Oort cloud and the outward migration of Uranus and Neptune imply the removal from that region of about $50 M_{\oplus}$ of condensed material, which is comparable to the core masses of all four giant planets combined. Thus a more realistic minimum mass would be about twice the classical value, which, however, is still insufficient according to the recent simulations. The discussion of an acceleration mechanism is the main point of this paper. We wish to show that even in a MMSN it could be possible to form Jupiter in the required time, just to show how efficient the mechanism proposed in this paper can be.

1.1. Formation of vortices

The vortices can either form primordially by perturbations induced by matter infalling onto the disk (Barranco & Marcus 2000), or result from a hydrodynamical instability, such as the Rossby wave instability (Lovelace et al. 1999; Li et al. 2001) or a global baroclinic instability (Klahr & Bodenheimer 2003a,b). Entropy gradients in rotating systems can produce Rossby waves, which can eventually break into vortices, as is known to occur near planetary surfaces in the solar system, and in the atmospheres of giant planets.

Until recently anti-cyclonic vortices were thought to be excluded by the action of magnetohydrodynamics (MHD) and that the magnetorotational instability (MRI) would destroy any large scale vortices. Recent simulations by Fromang & Nelson (2005) showed the opposite. They performed three-dimensional global MHD simulations of protoplanetary disks in a cylindrical potential, very similar to previous studies by Armitage (1998), Hawley (2001) and Steinacker & Papaloizou (2002). But Fromang & Nelson (2005) measured for the first time the vorticity distribution of the flow and found that large-scale and long-lived anti-cyclonic vortices formed. These extended vorticity minima are easy to miss as the short-lived and

small-scale vorticity fluctuations are stronger in amplitude. They also found that up to 75% of all meter sized objects (referred to here as “boulders”) randomly placed into the disk radially outside of the vortex were captured in those vortices for a simulation which ran for a time of 200 orbits at the inner edge of the disk. Less than half the boulders got lost and migrated off the grid, potentially into the star. We will again refer to this very important result for the discussion of the capture probability of boulders by vortices.

1.2. Stability of vortices

Two-dimensional anti-cyclonic vortices can survive in protoplanetary disks without strong turbulence, that is, with low viscosity, for many orbits (Godon & Livio 1999a,b). The analytic solutions for large three-dimensional vortices (Goodman, Narayan & Goldreich 1987) have recently been numerically tested for stability by Johansen, Andersen & Brandenburg (2004). They find that the lifetime of the 3-D vortex is inversely proportional to an artificially imposed background viscosity.

Recently, Barranco & Marcus (2005) studied the stability of three-dimensional vortices in stratified disks using an anelastic approximation. They find that an analytic vortex solution to the anelastic equations is not hydrodynamically stable if placed in the midplane of a disk. But these small subsonic¹ vortices are quite different from the large azimuthally extended vortices that show up in Klahr & Bodenheimer (2003a) and in Fromang & Nelson (2005). The vortex used by Barranco & Marcus (2005) was placed into the midplane, but it decayed and reappeared in the stratified atmosphere away from the midplane. This indicates that stratification is actually beneficial for the stability of the vortex. In fact a vortex seems to be stable if it is flat, e.g. the vertical extent is smaller than the radial and horizontal extent. Stratification is such an agent to keep vortices flat. Another possibility to allow for flat vortices is to use global models beyond the limits of a shearing sheet box. Fromang & Nelson (2005) showed that a three-dimensional vortex does survive in a global non-stratified disk. Thus, it is possible that the instability in Barranco & Marcus (2005) results from the limitations of the anelastic and of the local approach, while global and fully compressible hydrodynamics and MHD indicate a certain stability for three-dimensional anticyclonic vortices.

¹The anelastic approach can only handle subsonic flows.

1.3. Vortices in planet formation

It has already been suggested that anti-cyclonic vortices can be beneficial for the planetary formation process (Adams & Watkins 1995, Barge & Sommeria 1995; Tanga et al. 1996; Godon & Livio 1999b). In general the vortices can be regarded as preferred formation sites of planets because they tend to collect solid particles from the surrounding disk. If one invokes a three-phase model for the planetary formation process (Klahr 2003; Klahr & Bodenheimer 2003b,c) one can regard the formation of vortices as phase one. Phase two is characterized by the accumulation of solids in the center of vortices and by the growth of a planetary core, and phase three by the accretion of gas onto the core. In this paper we investigate the latter two phases in the framework of the core accretion model (Pollack et al. 1996). This model allows us to quantify the reduction in formation time of Jupiter by accelerated solid accretion into vortices, a radial expansion of the feeding zone, and a relatively efficient process of emptying the feeding zone.

We do not claim that planetesimals cannot form without a vortex, but vortices accelerate the planetesimal formation. For giant planets there is a time scale problem, to form the entire planet before the disappearance of the gas, which can be overcome by this mechanism. For terrestrial planets that particular time scale problem does not exist, and we do not deal in this paper with the numerous other problems of terrestrial planet formation.

Section 2 gives an overview of the current formation scenarios under discussion and shows how the new scenario fits into the old framework. In Section 3 we derive the accretion rate of solid material as an input parameter for the core accretion model. This model is then analyzed, in Section 4, through numerical simulations, as a possible way to form Jupiter rapidly. Section 5 summarizes and discusses our results and gives an outlook for future work.

2. Planet Formation Models

We introduce the new three-phase planet formation scenario because we think that the current models under discussion have substantial physical problems. The new model is not a revolutionary approach, but more a merging of the fundamental ideas from the existing models plus some vortex theory from geophysical fluid dynamics. Before we explain the new model we briefly describe the existing models and elucidate in more detail what the difficulties are. Further discussion of these models can be found in a review by Bodenheimer & Lin (2002).

2.1. Disk Instability Model

Boss (1997, 2001) proposed a model in which a cool protoplanetary disk, with roughly 10% of the mass of the star, fragments as a consequence of its self gravity and spontaneously forms multiple giant planets within a few orbital periods. Mayer et al. (2002) showed, in a numerical simulation with simple assumptions regarding cooling, that fragments can form and survive if the Toomre Q parameter is low enough, in their case about 1.4. The fundamental problem with this model is that it needs an almost isothermal gas to allow for the collapse and to keep the Toomre parameter low. Gammie (2001, see also Rice et al. 2003) shows that the cooling time t_{cool} has to be smaller than an orbital period t_{orb} .

$$t_{\text{cool}} < \frac{1}{2} t_{\text{orb}}. \quad (1)$$

Radiative diffusion within a dust-rich disk as well as in a self-gravitating blob is too inefficient to allow for such high cooling rates. Also the recently invoked thermal convection can only slightly increase the efficiency of the cooling (Bell et al. 1997); they show that the maximum fraction of the energy carried vertically outward through a disk by convection is about 20%. Convection in itself does not result in energy loss from the disk; it only transports the heat to the surface where it still has to be radiated away. Even if strong convection would be able to transport energy in the interior of the disk (Boss 2004) the gas would still have to radiate at the surface as a black body, at a maximum rate proportional to T^4 , where T is the central temperature in the disk. The cooling time would then be longer than an orbital period for radii out to 33 AU:

$$t_{\text{cool}} \approx \frac{\Sigma c_v}{2\sigma} T^{-3} = \frac{\Sigma}{10^3 \text{ g cm}^{-2}} \left(\frac{T}{50\text{K}} \right)^{-3} \times 190 \text{ yrs.} \quad (2)$$

Here we used typical surface densities Σ and temperatures T from the Boss model (Boss 2001); c_v is the specific heat and σ is the Stefan-Boltzmann constant. Note that for even only slightly smaller temperatures the cooling time increases dramatically. We argue on the basis of this estimate that it is possible to form brown dwarf companions out of an extended circumstellar disk at large radii but not planets at the distances of Jupiter or Saturn. Basically one can conclude that the conditions needed for planet formation by gravitational instability are unlikely to occur in disks.

2.2. Core Accretion

In the other formation model (e.g. Pollack et al. 1996; Wuchterl, Guillot & Lissauer 2000), first a solid core of at least $5\text{-}10 M_{\oplus}$ is built up via successive collisions of planetesimals.

Starting from this mass the core can accrete gas from the disk. The gas accretion rate is determined by the cooling of the protoplanet. This means that as long as there are still planetesimals plunging into the planetary atmosphere and releasing their potential energy within, the planet contracts only slowly and can not accrete gas efficiently. Only after the mass of the gaseous envelope approaches that of the solid core can the contraction of the envelope take place fast enough to provide a runaway gas accretion. Even for massive disks with three times the solid surface density of the MMSN, it takes roughly 6 Myr to form Jupiter (Hubickyj et al. 2005), if the grain opacity in the planetary envelope is based on an interstellar size distribution. However, their calculations also show that by artificially stopping the accretion of solids at an early time after the core is established, or by reducing the grain opacity in the envelope of the protoplanet below the interstellar value, it is possible to start the runaway phase earlier. The reduction in grain opacity is consistent with the detailed calculations of Podolak (2003). Also a modest enhancement of solids in the nebula, above three times that in the MMSN, can reduce the growth time substantially.

This model contains another so far unsolved problem. Dust can efficiently coagulate to build boulders up to a size of tens of centimeters (Blum & Wurm 2000). Sedimentation and radial drift due to gas drag and the sub-Keplerian rotation of the disk provide high enough relative velocities for efficient capture of smaller particles. At the same time the impact velocities are low enough so that the surface of the boulder is not destroyed by sputtering and fragmentation (e.g. Leinhardt, Richardson, & Quinn 2000) faster than it is supplied with mass.

But by the time the boulder reaches meter size, it will drift with velocity up to 100 m s^{-1} at 1 AU (Weidenschilling & Cuzzi 1993), or 22 m s^{-1} at 7.5 AU (which is the case we consider in §3.2). At velocities above 10 m s^{-1} the boulder gets eroded by the many impacts of smaller grains due to the high relative velocity with respect to the smaller objects. If this would not be a problem on its own, there is another effect. At a velocity of 100 m s^{-1} the boulder will drift into the sun faster than it can grow to a large enough size to be beyond the regime of high radial drift velocities. Thus the mechanism for forming kilometer-sized planetesimals is still an open question.

To be fair, in general there is no strong observational evidence that distinguishes between core accretion models and gravitational instability models, except that the correlation of the incidence of extrasolar planets with metallicity of the host star (Santos et al. 2003; Fischer & Valenti 2005) does favor core accretion models somewhat.

2.3. The Role of Vortices

Vortices combine the previous models in the following sense: anti-cyclonic vortices stabilize mass concentrations in the protoplanetary disk, for example the blobs in the Boss model, against shear. In this case the stabilizing effect is not gravity but the action of Coriolis forces. The resulting flow is in a so-called geostrophic balance, between pressure effects and Coriolis effects. The vortices are stable flow features even without cooling, and they do not need self gravity to stay bound (Goodman et al. 1987). This is the same effect that stabilizes hurricanes on earth or the giant red spot on Jupiter.

We suggest three reasons why vortices may be important for planet formation. In contrast to the sub-Keplerian gaseous disk, (1) at least the centers of vortices orbit at the Keplerian rate, (2) they have no vertical shear, and (3) they have no radial shear and thus will not generate (MHD)-turbulence. The following subsections amplify these points briefly.

2.3.1. *Vortices move at Keplerian speed*

First one has to define the center of the vortex, which we take to be given by the maximum in pressure, and which we refer to as the “eye”. At a local pressure maximum the radial and azimuthal pressure gradients vanish. Thus the only forces determining the motion of the gas at the eye of the vortex are gravity and centrifugal force. Thus the eye must move on a Keplerian orbit and is not bound to the sub-Keplerian motion of the gas outside the vortex.

For the vortex to generate a local pressure maximum in the disk, it must exceed some minimum strength, or amplitude, in terms of vorticity. While all cyclones are low-pressure regions and all anticyclones high-pressure regions, just as in the earth’s atmosphere, a weak vortex will not generate a local pressure maximum in the radially falling pressure profile in a disk. However the anticyclones reported in the numerical simulations of Klahr & Bodenheimer (2003a) are between 30 % to 100 % higher in pressure than the ambient gas, and they have a chance to decouple from the gas with the eye on a Keplerian orbit.

What are the consequences of this? Once small solid particles have accumulated in the eye of the vortex and have grown to kilometer size, they will decouple from the gas and no longer actively be bound to the vortex eye by the vortical gas motion. However they also will be in Keplerian motion, along with the eye. Thus, as long as there are no additional effects scattering them out of the vortex eye, the planetesimals may well stay in co-orbit with the vortex for many, many orbits.

Simulations by Klahr & Bodenheimer (2003c) and Fromang & Nelson (2005) have shown that meter-sized boulders stay captured once they are inside a vortex for hundreds of orbits even if the overall disk flow is strongly turbulent. Yet it is still to be shown what happens to objects large enough to decouple from the gas flow. They might drift out of the vortex due to some residual non-Keplerian drift of the vortex or even be scattered out of the vortex by the gravitational torques of the disk gas (Fromang & Nelson 2005). The fate of kilometer sized and larger objects definitively needs further investigation.

2.3.2. *Vortices have no vertical shear*

The vertical shear in an accretion disk is purely an effect of the stronger sub-Keplerian motion of the gas in the midplane than in the upper layers of the disk, because the pressure is the highest in the midplane and thus the radial pressure support the strongest. In contrast, all the way through the vertical extent of the vortex eye, the gas will move at the Keplerian frequency. Of course this vertical direction is not the vertical direction of a cylindrical coordinate system but is given by the local effective gravity in the system co-rotating with the vortex eye, which means that the rotational axis of the vortex bends slightly towards the rotational axis of the accretion disk with increasing height above the midplane. In a thin accretion disk this effect may well be unobservable.

The effect of the non-existence of a vertical shear is very interesting. It means that solids can sediment to the midplane and concentrate to a density higher than the critical value where self-gravitational effects become important, without generating a shear layer instability (Cuzzi et al. 1993).

2.3.3. *Inside a vortex there is potentially no (MHD)-turbulence*

As is known for hurricanes on earth, the eye of a (anti-) cyclone is quiet. There is probably little or no turbulence acting in the center of a vortex, because shear is required for the generation of, for instance, the magnetorotational instability. This radial shear is not present in a giant vortex. The vortices are also likely to be in a thermodynamical equilibrium owing to their relatively small dimensions, so that a baroclinic instability is also unlikely.

As a result of this absence of turbulence there will be very small RMS velocities between the boulders. Also, collisions will be gentle, and the likelihood of scattering out of the vortex is small.

It was already suggested that vortices could be the direct precursors of planetary forma-

tion. The planets could form either by concentration of dust in the centers of the vortices, as was suggested by Barge & Sommeria (1995), or by sufficient gas accretion onto a vortex so that it undergoes gravitational collapse (Adams & Watkins 1995). The latter possibility seems unlikely in the light of our observation (Klahr & Bodenheimer 2003a) that the vortices are still far away from a critical Jeans mass.

Basically we think that there are three possibilities concerning what happens to the solids once they are captured in the eye of the anti-cyclone.

1. The naive picture assumes that all captured solids will contribute to one single growing core. This picture has the possibly significant problem that the core might actually leave the vortex once it grows to kilometer size and decouples from the gas. Even though we stated that a strong geostrophic vortex will orbit at the Keplerian rate, as will the kilometer-size planetesimal, there are two sources of danger. First, the vortex is a dynamical feature, and it could migrate in the radial direction by interaction with the ambient disk. Second, even when the core forms from material with basically the same angular momentum as the vortex eye, a small variation in the specific kinetic energy in the azimuthal direction can lead to a slow azimuthal drift of the core out of the eye of the vortex. This problem might be overcome once one starts to investigate the feedback of the core on the gas, via gravity as well as via friction. These effects might stabilize once more the gas around the core.
2. In a second model one can assume that the boulders (meter size) that accrete into the vortex interior do not accumulate in the center and form one giant core, but that they form a “core zone”, enriched in solid mass but still containing some gas. This particle layer could then eventually undergo gravitational collapse (Goldreich & Ward 1973), which in this case will not be prevented by vertical shear. The precise conditions for this instability will have to be derived elsewhere; in particular, the velocity dispersion of the accumulated particles has to be investigated in detail. A similar study for solids immersed in local MHD turbulence has been performed by Johansen, Klahr & Henning (2006), who discuss the possibility of gravoturbulent formation of planetesimals in small-scale short-lived vortices. This study should be expanded to a global simulation. This picture has the benefit that all boulders might stay actually captured by the vortex until the core forms in one single collapse. Thus even if the vortex is not 100 percent precisely Keplerian, or if it radially migrates. the solids would follow the vortex and not get ejected.
3. In any case we do not expect the physical capturing process to be perfect. Thus probably a smaller or larger fraction of planetesimals that have decoupled from the gas

may get ejected from the vortex. If this is a minor effect, then this is a wonderful way to produce planetesimals in vortices and then scatter them to other regions of the disk, where they could be used in the formation of other planets, that form independently of a vortex.

If this scattering of the planetesimals out of the cyclone is more the rule than the exception, then it will become unlikely to form a planetary core mass inside the vortex. Nevertheless the process would produce 100 m to 1 km planetesimals which are difficult to form by any other means because of the effects of gas drag. Once scattered out of the vortex, the planetesimals will stay at about the same radius. These planetesimals, whose total mass would be $10\text{--}20 M_{\oplus}$ or more, would thus still be in a radially confined and strongly enriched feeding-zone and could accumulate to a core by collisions and gravitational focussing, as is assumed in the classical picture (Pollack et al. 1996). Thus the formation of a core for a giant planet is more likely at the radial position of a vortex, as the formation time for the core becomes reasonably small. Thus even if a planetary core does not form in a vortex, the presence of the vortex may be very beneficial for planet formation.

The third idea will have to be elaborated elsewhere. For this paper we will follow pictures one and two or an arbitrary mixture of both. For our model it is not important how the core forms, as long as the mass of the solids inside the core zone starts to gravitationally act on the surrounding gas in the eye of the vortex. Thus we simply assume that sooner or later the accreted solids will provide the potential well for the giant planet formation.

We propose that the formation of planets is probably characterized by three phases, that depend directly on each other:

- Phase 1: Formation of anti-cyclonic vortices as pre-protoplanetary condensations
- Phase 2: Accumulation of solids into the vortices to form protoplanetary cores
- Phase 3: Accretion of gas onto the protoplanetary cores

The following section will quantify phase 2, while section 4 will make predictions on both phases 2 and 3.

3. Accretion Rate of Solid Material

The accretion rate of solid material \dot{M}_s onto the planetary core depends on the available mass M_s in solid material in the feeding zone of the planetary embryo, the time scale for

particle production τ_p , the time scale for radial drift τ_d , and the capture probability q_c .

$$\dot{M}_s = \frac{q_c M_s}{\max \{\tau_d, \tau_p\}} \quad (3)$$

Whatever time scale is longer determines the accretion rate.

The capture probability q_c has shown to be between 50% and 75% (Fromang & Nelson 2005; see also Klahr & Bodenheimer 2003c). The high capture rate can be explained in the following way. The vortex is not a completely isolated entity in the disk. Even if it stretches only over 45 to 60 degrees in the azimuthal direction, the rotational profile is also changed in the rest of the disk. The remaining 300 degrees of the disk at the radial location of the anti-cyclone can be interpreted as a large and thus weak cyclone. This cyclone expels particles and therefore hinders their inward passage. It eventually transports them towards the anti-cyclone. It can be observed in the relevant simulations (Fromang & Nelson 2005) that particles randomly distributed in the disk radially outside of the vortex first drift to the radial location of the vortex and then drift azimuthally towards the center of the vortex. In the following we simply use the symbol M_s to represent $q_c M_s$. The consequence is that the minimum mass disk required to form Jupiter will be increased by q_c^{-1} which is probably less than 2.

3.1. Mass of Solid Material

In the standard core accretion model only mass in the vicinity of the core, in an annulus extending outward in both directions from the planet to about 4 Hill sphere radii, can be swept up via gravitational attraction and accreted onto the core. The Hill sphere radius is

$$R_H = R \left(\frac{M_p}{3M_*} \right)^{\frac{1}{3}} \quad (4)$$

where R is the distance to the central star and M_p and M_* are the masses of the planet and the central star, respectively. The value during planet formation is relatively small, leading to the need of a high surface density in solids and gas for the disk. It also grows only slowly with the mass of the planetary embryo, which makes the initial growth from less than an earth mass a long process. The formation process is also held up by the slow contraction rate of the gaseous envelope, once the core has swept up most of the available solid material. Models without an anti-cyclonic vortex need a surface density in solid material about three times that of the MMSN to form Jupiter at 5 AU in less than 10^7 years, which is the maximum accepted lifetime of the nebula itself.

If there are anti-cyclonic vortices in the disk, then the accumulation of solids is no longer driven by gravity but by gas drag. Thus, the reservoir of solids that can be accreted is not given by the Hill radius, but simply by the radial separation of the vortices in the solar nebula. Simulations (Klahr & Bodenheimer 2003c, Fromang & Nelson 2005) have shown that mobile solids are accreted into their closest inner vortex in a short time at a 50-75 % percent efficiency. For this paper we consider the inner (Jupiter) vortex to be located at 5 AU and the outer (Saturn) vortex at 10 AU. We adopt a reasonable solar nebula model (Bell et al. 1997) with a total surface density of 250 g cm^{-2} on the average from 5 AU out to 10 AU. These parameters correspond to a disk with an effective viscosity of $\alpha = 10^{-3}$ and a mean accretion rate of the disk gas onto the star of $\dot{M} = 10^{-8} \text{ M}_\odot \text{ yr}^{-1}$. With these parameters one is close to the MMSN and also in good agreement with the currently accepted accretion disk theory. For a conservative solid to gas fraction of 1 : 100 this corresponds to a surface density of $\Sigma_d = 2.5 \text{ g cm}^{-2}$ in solid material. The total solid mass between 5 and 10 AU is then:

$$M_s = 2\pi \Sigma_d R_{mean} dR = 1.33 \times 10^{29} \text{ g} = 22M_\oplus. \quad (5)$$

This is enough mass to form a core for Jupiter, even if the growth and capture processes are not 100 percent efficient, and also to explain its mean metallicity. If the capture probability is as low as 50 % (Fromang & Nelson 2005), then one can still compensate for this effect by a two times larger mass in the nebula, which is still less than the 10 times MMSN required by previous work (Weidenschilling 1998; Inaba, Wetherill, & Ikoma 2003; Thommes, Duncan, & Levison 2003).

3.2. Particle Drift

The radial drift velocity v_r of particles with friction time τ_f sets a lower limit to the aggregation time of a core in the anti-cyclonic vortex:

$$\tau_d = dR/v_r(R_{mean}). \quad (6)$$

The actual time a particle of radius $a \sim 30 \text{ cm}$ needs inside the vortex to spiral to its center is only one orbital time (10 yr) (Chavanis 2000) and thus can be neglected, as we will see later. Chavanis (2000) also calculated the escape probability due to stochastic motion for the boulders and found that 30 cm particles have no chance to escape from the vortex in the time needed for the planet formation process.

The radial drift velocity for the relevant objects, up to meter size, is given by Weidenschilling & Cuzzi (1993):

$$v_r = 2dV\Omega\tau_f, \quad (7)$$

where Ω is the orbital frequency and dV is the deviation of the rotational velocity of the gas from the Keplerian value, which is an effect of the radial pressure gradient in the gas. This deviation can be estimated to be

$$dV = \frac{c_s^2}{\Omega R}. \quad (8)$$

The sound speed at $R_{mean} = 7.5AU$ in our model is $c_s = 4 \times 10^4$ cm/s, and the Keplerian velocity is $V_{Kep} = 1.1 \times 10^6$ cm/s. Thus the deviation from Keplerian velocity is

$$dV = 1.5 \times 10^3 \text{ cm/s}. \quad (9)$$

The dimensionless friction time ($\tau_f \Omega$) for particles of radius a and density ρ_d in the Epstein regime is determined by

$$\tau_f \Omega = \frac{a \rho_d}{c_s \rho} \Omega \approx \frac{a \rho_d}{\Sigma}, \quad (10)$$

for gas of density ρ and surface density Σ . The maximum drift velocity will occur for $\tau_f \Omega = 1$ particles with $\rho_d = 1.39 \text{ g/cm}^3$ at the size of 180 cm. We can then define:

$$v_r = dV \frac{a}{90\text{cm}} \text{cm/sec}, \quad (11)$$

which holds for boulders of up to 90 cm in radius. The drift time for $dR = 2.5 \text{ AU}$ is a function of particle size:

$$\tau_d = 8 \times 10^2 \frac{90 \text{ cm}}{a} \text{ yr}. \quad (12)$$

So even for bowling-ball-sized boulders ($a = 10.8 \text{ cm}$) the radial drift time is less than 8000 yr. In Figure 1 we compare the radial drift time as a function of particle size to the characteristic growth time.

3.3. Particle Growth

The weakest point of our estimate is the growth time τ_p for 1 cm to 1 m sized boulders starting from micron sized dust, or, to be more precise, the production rate of those objects $\dot{M}_p = M_s / \tau_p$. Smaller objects do not decouple sufficiently from the gas to reach a vortex in less than the lifetime of the protoplanetary disk. Based on simulations of particle growth (e.g. Weidenschilling & Cuzzi 1993) and experiments by Blum & Wurm (2000) we assume the following scenario: particles grow first by Brownian motion which takes only a couple of thousand years at 7.5 AU. This initial growth can occur while the vortex at 5 AU is still forming, which also will take a few times 10^4 years (Klahr & Bodenheimer 2003a). Afterwards particles sediment and grow owing to sedimentation, which is also a fast process. Then they drift radially inward and sweep up smaller particles until the collision velocity

exceeds the fragmentation velocity of about 10^3 cm/s. This part of the growth takes the longest time, and thus we neglect the time used in the previous stages. During the radial drift, the mass gain per boulder \dot{m} is given by

$$\dot{m} = v_r \sigma_d n_s m_s. \quad (13)$$

This means the accumulation of mass \dot{m} is proportional to the relative velocity v_r between the boulder and smaller particles, the cross section of the boulder σ_d , the number density n_s of the smaller particles in the path of the boulder and their mass per particle m_s . This expression can be simplified and leads to a differential equation for the particles' mass (m) as a function of the reference particle size $a_0 = 90$ cm and of the ratio between the density of the boulder ρ_d to the density of the small dust material $\rho_s = \Sigma_s/H = 6 \times 10^{-13}$ g cm $^{-3}$ in the gas:

$$\dot{m} = m \left(\frac{6dV}{4a_0} \frac{\rho_s}{\rho_d} \right). \quad (14)$$

This equation indicates exponential growth with the typical time τ_g at 7.5 AU of

$$\tau_g = \frac{90}{1.1 \times 10^3} \frac{\rho_d}{\rho_s} \text{sec} = 6 \times 10^3 \text{yr}. \quad (15)$$

The entire growth time τ_e for particles from micron to 10 cm sized objects, e.g. fifteen orders of magnitude in mass, should be about $\tau_e = 10^5$ yr, as a rough estimate with the assumed physics. This gives only an upper limit to the growth time. Any settling would decrease the growth time.

From Figure 1 we see that the drift time scale τ_d is shorter than the growth time scale τ_g for particles larger than 10 cm. This means after they reach a size of 10 cm they will drift into the next vortex before they can grow by an order of magnitude in size. Thus, the boulders grow on the average in $\tau_e = 10^5$ yrs to the critical size, and then they become accreted within 4×10^3 yr into the center of the vortex to form the core.

It is more complicated to estimate the production rate $\sim \tau_p$ of 10 cm solids. Theoretically, all boulders could grow simultaneously and reach the critical size at the same time. Then the production rate would be infinite and the drift time scale the only limiting factor. On the other hand there is a spread in the particle size distribution (see Weidenschilling & Cuzzi 1993). This translates into a spread in the time that particles need to grow to the critical size of maybe $\pm \frac{1}{2} \tau_e$. Thus, we can conservatively estimate an accretion rate of solids of $\dot{M}_s = 2.2 \times 10^{-4} M_{\oplus}/\text{yr}$. In case our estimate is still wrong by one order of magnitude we also consider a model of $\dot{M}_s = 2.2 \times 10^{-5} M_{\oplus}/\text{yr}$ and compare the results.

It is interesting to note that the drift time will be smaller than the growth time for particles of about $a = 10$ cm (see Fig. 1), which means that no significant number of larger

particles can probably form outside the vortices and thus, the critical velocities for fragmentation of particles which occurs for $a = 90$ cm sized boulders will not be reached. This fragmentation was so far considered to be the end of planetesimal growth (e.g. Leinhardt, Richardson, & Quinn 2000), and the formation of larger objects an unsolved problem. We conclude that vortices are not only helpful to form gas giants but also to build up planetesimals, which will be either scattered out of the vortex later on or set free after the vortex has dispersed.

4. Time Scales for Gas Accretion

The assumed mechanism for planet formation is the core accretion – gas capture process. The vortex is assumed to have been formed at 5 AU from the central object. Particles in the 10 cm size range migrate inward in the disk as a result of gas drag and accumulate in the vortex where they quickly spiral toward its center. The vortex is assumed to last long enough so that all the solid particles originally between 5 and 10 AU are captured by it.

The procedure for numerically calculating the formation of a planet is described by Pollack et al. (1996). There are three main elements of the calculation: 1) the determination of \dot{M}_s by accretion of planetesimals, 2) the calculation of the interaction of accreting planetesimals with the gaseous envelope through which they fall, to determine where they dissolve in the envelope or whether they reach the core, and 3) the calculation of the evolution of the envelope. Steps 1 and 2 are simplified in the current calculations.

Although Pollack et al. (1996) made a detailed calculation of the variable accretion rate of planetesimals onto a planetary core, in this case we simply assume that small particles are accreted at a constant rate by the vortex, and that they quickly fall to the center to join the planetary core. The accretion rate stays constant until all the available solid material has been accreted by the vortex, after which time \dot{M}_s is set to zero. This last assumption also differs from those of Pollack et al. (1996) in that they assume that solid accretion continues even during the gas accretion phase. In our picture there are no further solids available to accrete. The core is set to a constant density of 3.2 g cm^{-3} .

Case	$\dot{M}_s(M_\odot \text{yr}^{-1})$	$M_{\text{core,final}}(M_\oplus)$	Opacity	Formation Time (yrs)
I	2.2×10^{-4}	22	high	0.3×10^6
II	2.2×10^{-5}	15	high	1.3×10^6
III	2.2×10^{-5}	15	low	0.8×10^6

Table 1: Parameters chosen in the different models and resulting formation time for Jupiter.

The planetesimal interaction with the gaseous envelope is not calculated. Planetesimals are assumed to fall through the envelope and hit the core, or, if they become dissolved in the envelope, to settle to the core. These assumptions do not give results significantly different from those of Pollack et al. (1996). Thus the energy liberated at the boundary of the core by planetesimals falling onto it is approximately

$$L_{\text{acc}} = \frac{GM_{\text{core}}\dot{M}_{\text{s}}}{R_{\text{core}}} \quad (16)$$

The calculation of the envelope is carried out through an implicit numerical solution of the spherically symmetric stellar structure equations as given by Bodenheimer & Pollack (1986). The energy sources are planetesimal accretion and gravitational contraction of the gas. The outer edge of the planet at a radius R_p is assumed to be the smaller of the Hill sphere radius (eq. 4) or the Bondi accretion radius

$$R_b \approx \frac{GM_p}{c^2} \quad (17)$$

where c is the sound speed in the disk just outside the planet. The surface boundary conditions at R_p are taken to be a constant density and temperature taken from a standard disk model (Bell et al. 1997). In the cases presented here the outer temperature $T_{\text{neb}} = 150$ K, and the outer density $\rho_{\text{neb}} = 5 \times 10^{-11}$ g cm $^{-3}$. The mass accretion rate for the envelope \dot{M}_{gas} is determined by the requirement that $R_p = \min(R_H, R_b)$. These radii expand as the planet gains mass, and especially at later stages the planet tends to contract, so mass is added until the above requirement is satisfied.

The equation of state of the envelope, which is assumed to have solar composition, is provided by the tables of Saumon, Chabrier, & Van Horn (1995). The opacities are provided by grains at low temperatures, and by molecules and other sources at temperatures above about 1800 K, where the most refractory grains evaporate. The highest temperatures that are encountered are in the range 20,000 K. The Rosseland mean opacities in the range 800–20,000 K are provided by the table of Alexander & Ferguson (1994), while those in the lower temperature range are obtained from Pollack, McKay, & Christofferson (1985). These opacities in the grain regime are based on the assumption that they have an interstellar size distribution. In the absence of detailed calculations, we assume that the grains in the vortex in the gas outside the planetary core have those properties. However we do one calculation in which the opacity in the grain regime (only) is reduced by a factor of 100. This assumption is justified by the work of Podolak (2003), who did a calculation of grain coagulation, settling, and evaporation in the envelope of a protoplanet, redid the opacity calculation on the basis of the modified size distribution that he found, and concluded that the opacities are roughly two or more orders of magnitude reduced from the interstellar values.

The initial condition for the simulation of the evolution of the envelope is taken to be a core mass of $1 M_{\oplus}$. The hydrostatic solution for the value of the envelope mass at that point gives a value of $1 \times 10^{-4} M_{\oplus}$. During the course of the calculation the envelope mass remains small until the core mass has reached its limiting value, after which time the envelope mass grows at an accelerating rate. When the envelope mass equals the core mass, the rapid gas accretion phase begins, during which the total mass builds up to the final planet mass, as determined by disk clearing or gap formation, on a time scale of only 10^4 yr. Since the main point of the calculation is to determine the formation time of the planet, we stop the calculation when the envelope mass is slightly larger than the core mass. We note that a standard core accretion process, without the presence of the vortex, in a disk at 5 AU with a solid surface density of 2.5 g cm^{-2} , would have a formation time much longer than the typical half-life of a disk, 3 Myr (Haisch et al. 2001).

The parameters of the three cases are listed in Table I. Case I is the standard case discussed above. In Case II the assumed accretion rate is decreased by a factor of 10, and it is assumed that the final core mass is only $15 M_{\oplus}$. Both of these changes tend to increase the formation time, so these parameters give a rather unfavorable case for planet formation. Case III is identical to Case II, except the grain opacities are reduced by a factor of 100. This change tends to speed up planet formation during the phase of relatively slow gas accretion (Pollack et al. 1996).

As discussed above the evolution separates fairly neatly into a core accretion phase and an envelope accretion phase. The masses of the core and envelope, as well as the luminosity radiated at the planet's surface, are plotted as a function of time for Case I in Figure 2. During the first 20000 yr the value of \dot{M}_s is gradually increased from an initial value of $1.4 \times 10^{-5} M_{\oplus} \text{ yr}^{-1}$ to its final constant value of $2.2 \times 10^{-4} M_{\oplus} \text{ yr}^{-1}$, which it reaches when the core mass is $2 M_{\oplus}$. The luminosity at this point is $10^{-5} L_{\odot}$. Thereafter the core mass grows linearly with time, the envelope mass is negligibly small, and the luminosity grows as $M_{\text{core}}^{2/3}$. Starting at about 10^5 yr, \dot{M}_s is smoothly reduced to zero. The envelope mass at this time is about $0.2 M_{\oplus}$. As the energy source arising from accretion of planetesimals is cut off, the luminosity rapidly falls, since the only remaining source is the gravitational contraction of the low-mass envelope. The luminosity levels off at just above $10^{-6} L_{\odot}$. As the envelope contracts rapidly to supply the radiated luminosity, its mass increases according to the requirement that the outer edge of the planet be at R_p . As the envelope mass increases, the luminosity increases as well. At about 3×10^5 yr the core mass and envelope mass are the same, the so-called crossover mass. Thereafter the envelope mass increases extremely rapidly until gas accretion is terminated by gap opening or by the dissipation of the disk. This later stage is not followed because the total formation time will be only slightly longer than the time to crossover. Essentially planet formation is complete at 3×10^5 yr, a factor

of 20 faster than the standard core accretion model would need with a considerably higher (factor 4) solid surface density and with the same opacity.

A plot of the same quantities is shown in Figure 3 for Case II. The core accretion rate is a factor 10 slower than in Case I and the accretion time for the core is 6×10^5 yr, less than a factor of 10 longer than in Case I because the final core mass is only $14.7 M_{\oplus}$ rather than $22 M_{\oplus}$. At a core mass of $2 M_{\oplus}$, the luminosity is $10^{-6} L_{\odot}$, a factor of 10 lower than that in Case I at the same core mass. Again the luminosity drops by about an order of magnitude just after the core accretion is cut off, then begins to climb again as the envelope mass approaches that of the core. The luminosity during most of the envelope accretion phase is about a factor 5 less than that in Case 1, mainly as a result of the smaller core mass, as explained in Pollack et al. (1996). Thus the contraction rate, and the accretion rate of the gas, are slower by a similar factor. The gas accretion phase lasts 7×10^5 yr, compared to 1.8×10^5 yr in Case I. The overall evolution time is 1.3×10^6 yr, a factor of 4.3 longer than that in Case I, but still shorter than typical disk lifetimes.

The only difference between Case II and Case III (Figure 4) is the reduction of the opacity in the envelope. The core accretion phase is unaffected, so the time scale and luminosity are the same as in Case II. The reduction in the envelope opacity results in an increase in the average envelope luminosity by about a factor of 3 in comparison with Case II. The envelope accretion phase lasts 2.2×10^5 yr, and the total formation time is 8.2×10^5 yr. Note that if the same opacity reduction had been applied to Case I, which has the standard core accretion rate, the overall evolution time would have been only 2×10^5 yr, which is the most likely estimate of the planetary formation time in a vortex, since it incorporates the most reasonable values of parameters.

5. Conclusion

We present a new formation model for gas giants. The general idea is that a giant vortex can accelerate the core formation considerably, even in a low-mass disk. The envelope accretion phase is speeded up also, because once the core has accreted all available solid material, the only energy source available for the gaseous envelope is its own contraction. The main reason for the long formation times for Jupiter in the earlier models of Pollack et al. (1996) was the additional contribution of planetesimal accretion to the energy supply of the envelope, during the first part of the gas accretion phase. We determine approximately, for the first time, the resulting time scales of such a scenario for the case of Jupiter's formation in a MMSN, (or twice MMSN, if the capture probability is as low as 50%). If a vortex had been responsible for the formation of Jupiter, the formation time would fall in the range

$2 \times 10^5 - 1.3 \times 10^6$ yr.

The major drawback of our model is that there are no direct observations of giant vortices in protoplanetary disks so far. But observations are planned to look for giant vortices with future instruments such as ALMA (Wolf & Klahr 2002). A vortex can be observed with ALMA because it is usually a region of higher gas surface density and thus also more dust is available to emit radiation. The boulders that become trapped in the vortices are not observable, but if they collide then some micron sized debris could enhance the dust to gas ratio and increase the opacity in the vortex region. But again, in Wolf & Klahr (2002) we show that the vortex can be observed even without accumulating more dust.

This paper concentrates on the second and third of the phases outlined in §2.3. It still needs to be proved that (1) conditions necessary for vortex formation actually commonly occur in disks, and (2) that a vortex actually survives long enough so that a planetary core of $10-20 M_{\oplus}$ can form in it. Initial studies by Li et al. (2001) and Klahr & Bodenheimer (2003a,b) and most recently for the case of MHD by Fromang & Nelson (2005) indicate that vortices can form, and work by Adams & Watkins (1995) and Johansen et al. (2004) shows that vortices can survive, at least up to several 10^4 yr at 5 AU (Godon & Livio 1999a,b). A linear stability analysis shows that vorticity can be generated from entropy gradients in the disk (Klahr 2004; Johnson & Gammie 2005), which is a necessary condition to form large scale vortices. Johansen & Klahr (2005) showed that in local MHD simulations one can find anti-cyclonic vortices to form and concentrate particles, but those small vortices which are part of the turbulent flow, do only survive a few orbits, before they decay. Even more convincing are the global simulations by Fromang & Nelson (2005) which find the formation of long-lived anti-cyclonic vortices in MHD turbulence. Thus, it seems that vortex formation is a generic feature to any kind of turbulence in accretion disks.

Further work is required to show how robust the vortex production process is. Assuming that the above conditions are satisfied, the main benefits of the vortex-core planet formation model are:

- No need for a solar nebula much more massive than minimum mass.
- Diminished loss of boulders as a result of drift into the central object.
- No fragmentation of boulders as a result of high impact velocities.
- Gentle aggregation of a core in the quiet eye of the vortex, which need not be self-gravitating.
- A formation time far less than the lifetime of the nebula.

We conclude that this model is able to solve outstanding problems in the theory of planet formation, and that further work on the difficult problem of vortex generation, through MHD simulations including radiation transport, transport and feedback effects of the solid boulders, and self gravity in three space dimensions, is warranted.

We want to thank our referee Stuart Weidenschilling for his constructive criticism, and Doug Lin and Norm Murray for fruitful discussions. This research has been supported in part by the NSF through grant AST-9987417, by NASA through grants NAG5-4610, NAG5-9661, and NAG5-13285, and by a special NASA astrophysics theory program which supports a joint Center for Star Formation Studies at NASA-Ames Research Center, UC Berkeley, and UC Santa Cruz. The work was supported in part by the European Community's Human Potential Programme under contract HPRN-CT-2002-00308, PLANETS.

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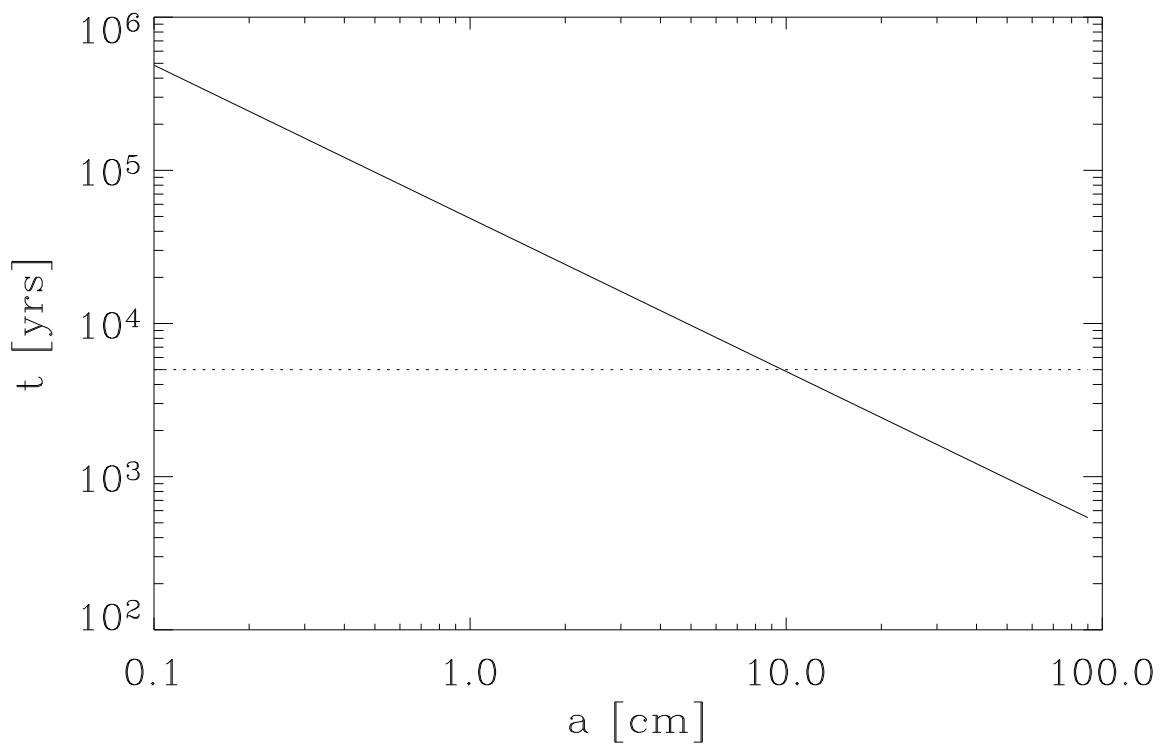


Fig. 1.— A comparison between drift time (*solid line*) and growth time (*dotted line*) for solids as a function of size. The values are calculated using the equations from this paper for a location of 7.5 AU in a minimum mass solar nebula.

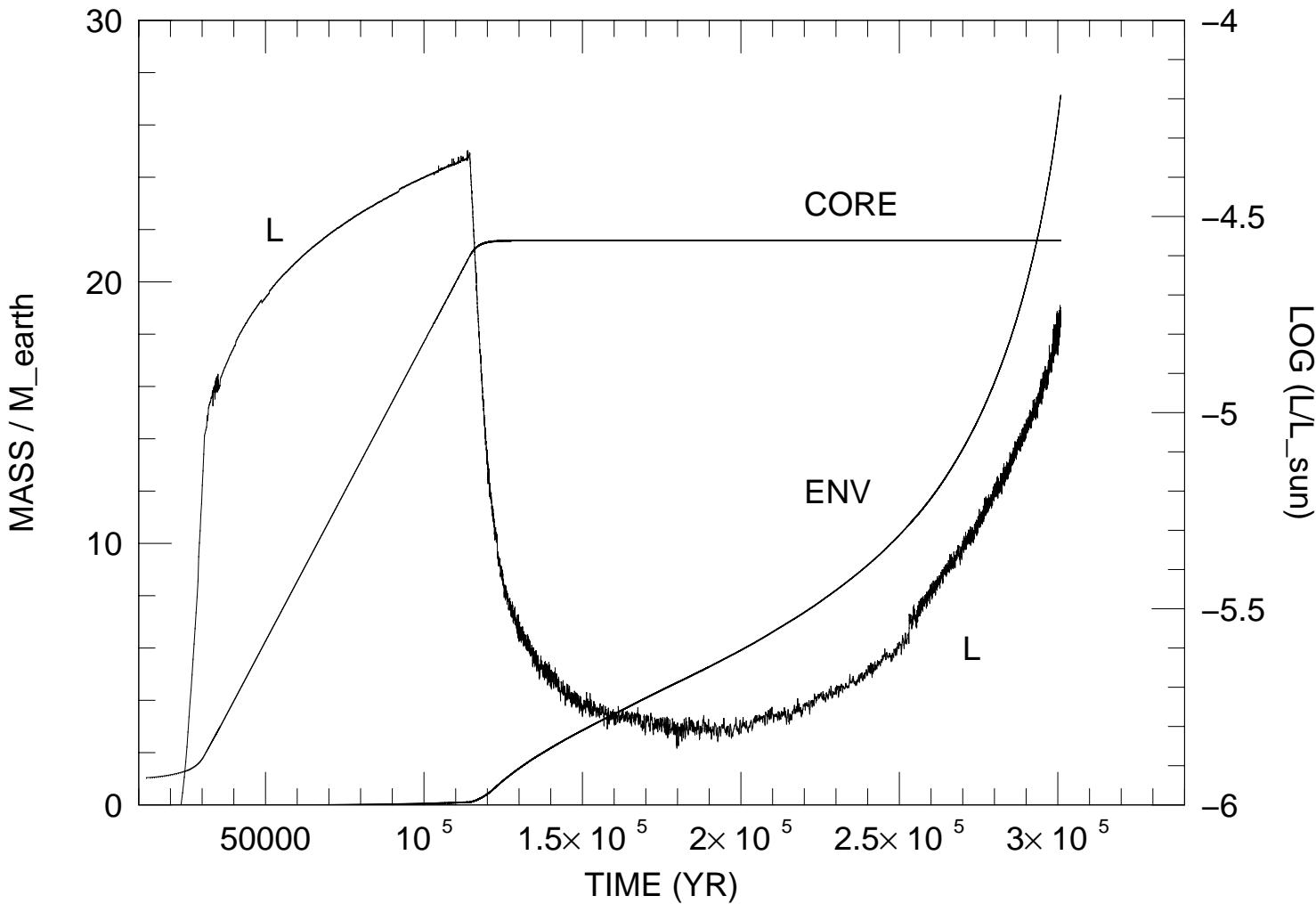


Fig. 2.— Case I: Evolution of the planet's luminosity (L), core mass (CORE), and envelope mass (ENV) as a function of time.

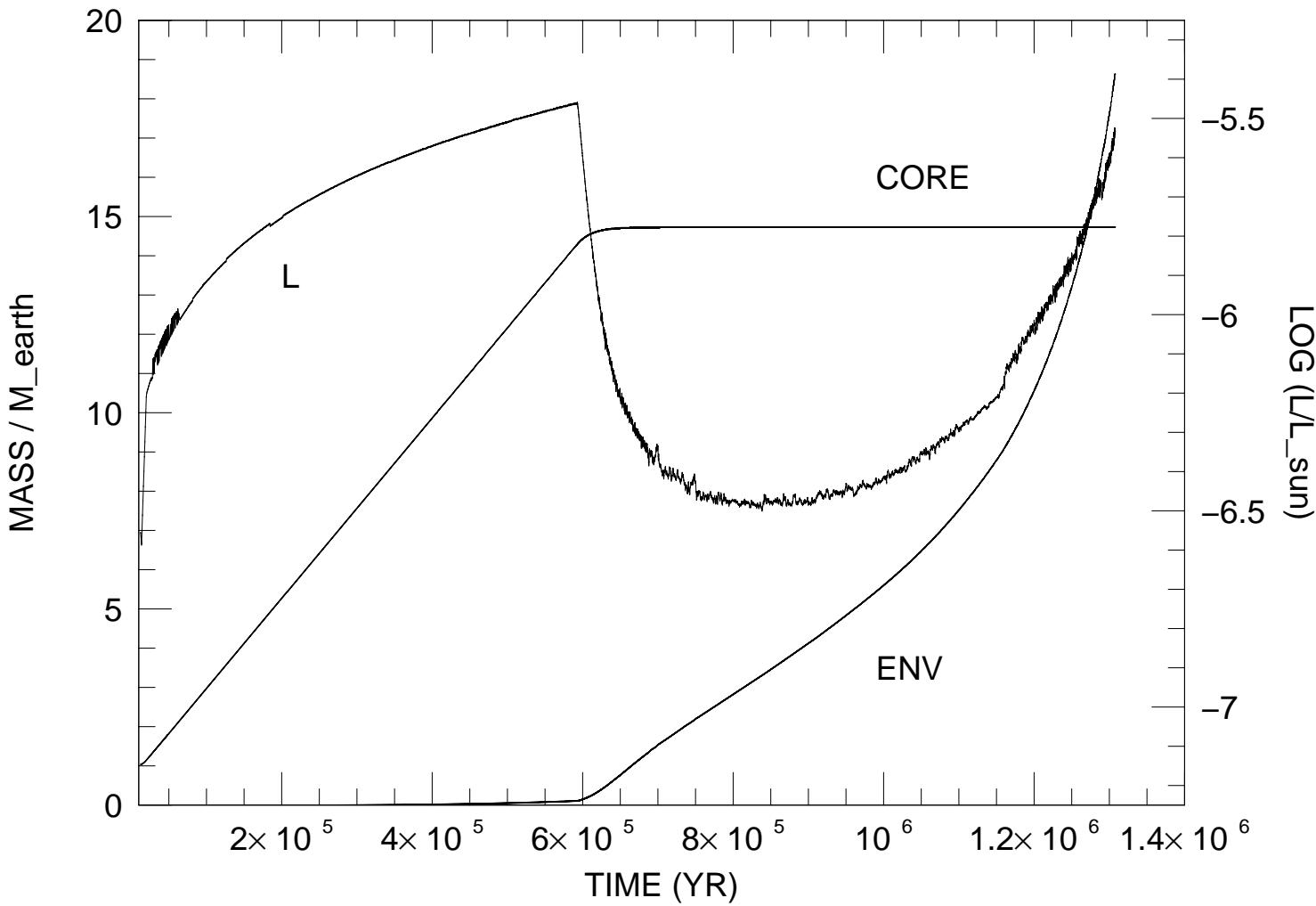


Fig. 3.— Case II: Evolution of the planet's luminosity, core mass, and envelope mass as a function of time. Labels have the same meaning as in Fig. 2.

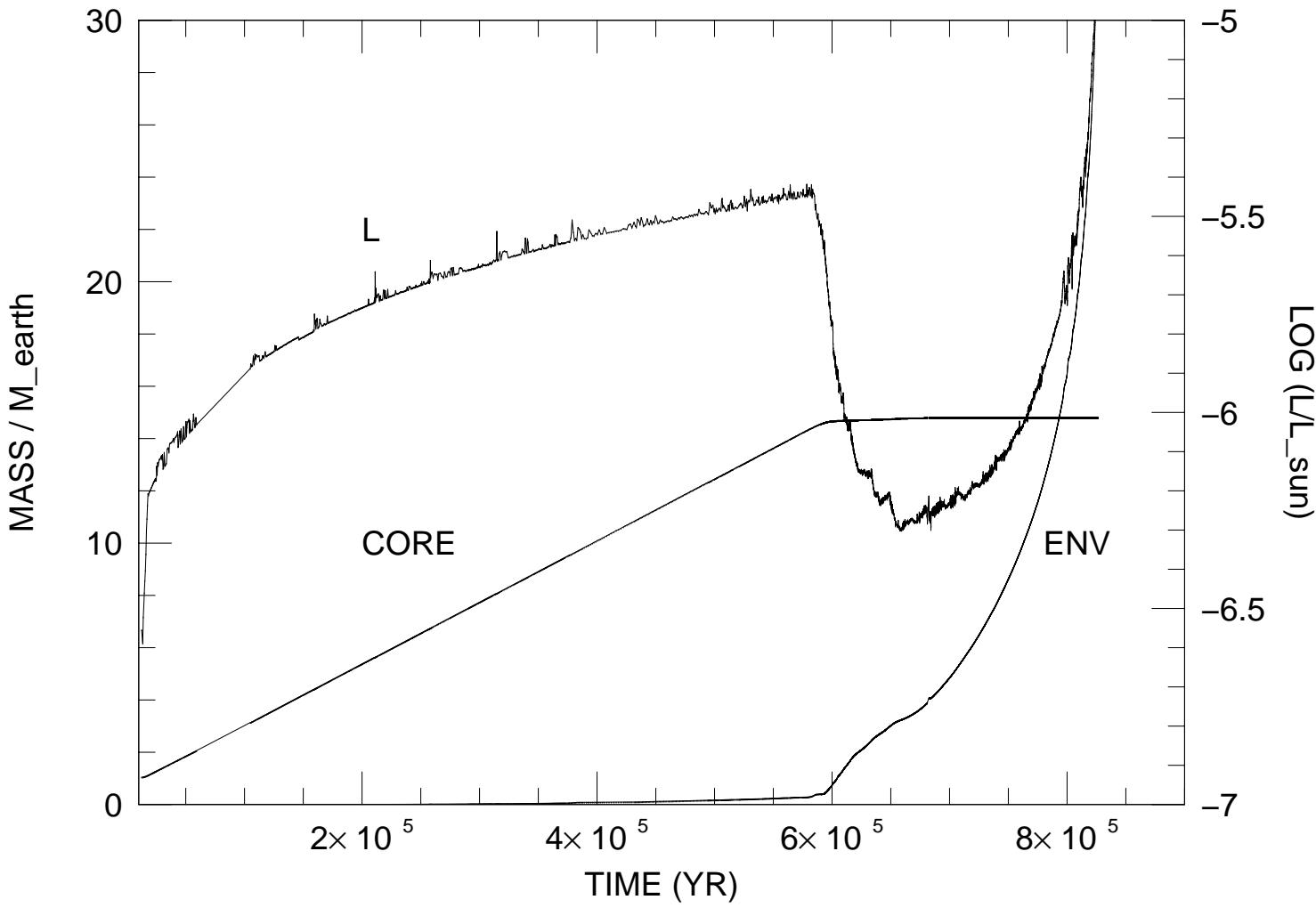


Fig. 4.— Case III: Evolution of the planet's luminosity, core mass, and envelope mass as a function of time. Labels have the same meaning as in Fig. 2.